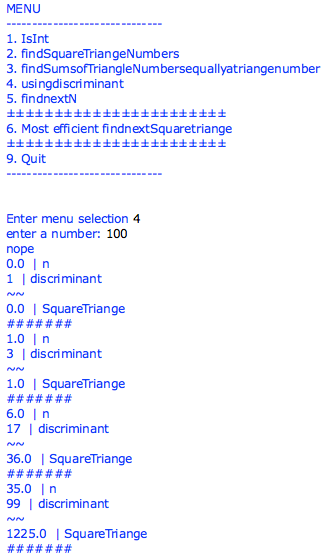
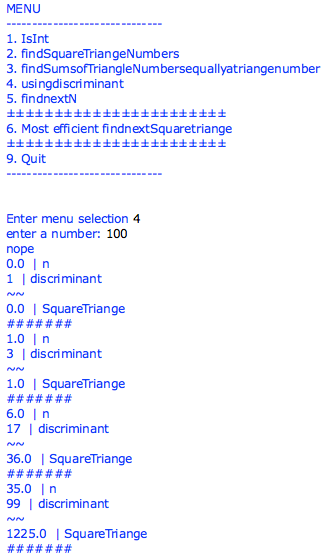
Establishing a pattern

Firstly, using the “usingdisciminant()” function in my program I looked at the values of n being presented:



The values of n produced here: 0, 1, 6, 35

After entering larger numbers to find more values of n I got a list:

n: 0,1,6,35,204,1189,6930,40391

I wanted to know if there were any pattern in these numbers. So I went in for differences:

0 1 6 35 204 1189 6930 40391

1 5 29 169 985 5741 33461

4 24 140 816 4756 27720

20 116 676 3940 22964

96 560 3264 19024

464 2704 15760

2240 13056

10816

At this point things looked alright, I pattern might still hold, but my values of n were getting bigger and bigger, and therefore taking longer and longer to calculate, so I decided to try so if I could do more with the numbers I did have, I took all the numbers on the left side of the triangle above, and used them as a base for the top row of a new triangle:

0 1 4 20 96 464 2240 10816

1 3 16 76 368 1776 8576

2 13 60 292 1408 6800

11 47 232 1116 5392

36 185 884 4276

149 699 3392

550 2693

2143

Again this looked fine, if not better than before but still a bit awkward so I did the same thing again: (i.e. *I took all the numbers on the left side of the triangle above, and used them as a base for the top row of a new triangle*)

0 1 2 11 36 149 550 2143

1 1 9 25 113 401 1593

0 8 16 88 288 1192

8 8 72 200 904

0 64 128 704

64 64 576

0 512

512

At this point I saw an obvious pattern on the left side of the triangle:

0 1 0 8 0 64 0 512

Is the same as:

0 20 0 23 0 26 0 29

In seeing this pattern and wanting to escape these triangles, I decided to assume that this pattern would hold (ie. If I assigned an ordinal number to each in the list above if odd, the number there should be 0, if even, the number should equal

1 2 3 4 5 6 7 8

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

0 20 0 23 0 26 0 29

)

Finding a way to use the pattern

Now in order for this to be useful I had to somehow relate these numbers

0 20 0 23 0 26 0 29

To these (the square triangles):

0 1 6 35 204 1189 6930 40391

So I decided to ignore exact values and look into how my pattern triangles worked algebraically: *(note I would have used some kind of subscript rather than all different characters, I felt this could have gotten more confusing when moving from one difference triangle to the next so I used unique characters for each position)*

|||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||

a b c d e

f g h i

j k l

n m

o

f=b-a

g=c-b

h=d-c

i=e-d

j=g-f

k=h-g

l=i-h

m=k-j

n=l-k

o=m-n

Some of these can be simplified as such:

j=c-2b+a

k=d-2c+b

l=e-2d+c

m=d-3c+3b-a

n=e-3d+3c-b

o=e-4d+6c-4b+a

Triangle 1

|||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||

a f j m o

p q r s

t u v

w x

y

p=f-a

q=j-f

r=m-j

s=o-m

t=q-p

u=r-q

v=s-r

w=u-t

x=v-u

y=x-w

Some of these can be simplified as such:

p=b-2a

q=c-3b+2a

r=d-4c+5b-2a

s=e-5d+9c-7b+2a

Triangle 2

t=c-4b+4a

u=d-5c+8b-4a

v=e-4d+6c-4b+a

w=d-6c+12b-8a

x=e-7d+18c-20b+8a

y=e-8d+24c-32b+16a

a p t w y

z aa bb cc

dd ee ff

gg hh

ii

z=p-a

aa=t-p

bb=w-t

cc=y-w

dd=aa-z

ee=bb-aa

ff=cc-bb

gg=ee-dd

hh=ff-ee

ii=hh-gg

Some of these can be simplified as such:

Triangle 3

z=b-3a

aa=c-5b+6a

bb=d-7c+16b-12a

cc e-9d+30c-44b+24a

dd=c-6b+9a

ee=d-8c+21b-18a

ff=e-10d+37c-60b+36a

gg=d-9c+27b-27a

hh=e-11d+45c-81b+54a

ii=e-12d+54c-108b+81a

|||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||

Now using the outcomes of the three triangles:

a=a

z=b-3a

dd=c-6b+9a

gg=d-9c+27b-27a

ii=e-12d+54c-108b+81a

So rearranging to get our square triangles = … :

a= a

b= z + 3a

c= dd + 6b - 9a

d= gg + 9c - 27b + 27a

e=ii + 12d - 54c + 108b-81a

The pattern of coefficients here is:

1 12 54 108 81

1 9 27 27

1 6 9

1 3

1

So overall, you can predict the co-efficients to the next letter from the top line of Triangle 1.

a=a

b=z+3a

c=dd+6b-9a

d=gg+9c-27b+27a

e=ii+12d-54c+108b-81a

1 4\*31 6\*32 4\*33 34

1 3\*31 3\*32 33

1 2\*31 32

1 31

30

So, to find ‘e’ we must know, a,b,c and d and ii.

* a,b,c and d are our previous square triangle numbers
* we can predict the value of ii using the ordinal value of e discussed at the end of the previous section:
  + if the ordinal is odd then: ii=0
  + if the ordinal is even then : ii=

e is the 5th number in our list of roots of square triangle numbers so ii = 0 as 5 is odd.

Formula to find values of n in the equation n2 =

Lets call the root of the nth square triangle number “STn ”, and “Value” be the predicted number based on the pattern seen at the end of the previous section.

So:

STn=Value-1()

Where Value

And means “(n-1) CHOOSE r“ as in the choose function

So now that I have a formula for STn all you have to do is keep a list of previous ST to generate the STn+1 . And to find your list of Square Triangle numbers, just square each number of the ST list.

0 1 6 35 204 1189 6930 40391

1st 8 terms of the ST list

This document should hopefully explain what I did in the functions:

* findnextn()
* findnextSquaretriangle()